

# Quark Mass Matrices from a Softly Broken U(1) Symmetry

**Ernest Ma**

*Physics Department, University of California, Riverside, California 92521*

## **Abstract**

Assigning U(1) charges to the quarks of the standard model, and allowing one extra scalar doublet with  $m^2 > 0$ , the correct pattern of the *up* and *down* quark mass matrices is obtained, together with their charged-current mixing matrix.

In the standard model of particle interactions, quark masses and the charged-current mixing matrix,  $V_{CKM}$ , which links the  $(d, s, b)_L$  quarks to the  $(u, c, t)_L$  quarks, are known to exhibit a hierarchical pattern [1].

$$\begin{aligned} m_u &\sim 1 - 5 \text{ MeV}, & m_d &\sim 3 - 9 \text{ MeV}, & m_s &\sim 75 - 170 \text{ MeV}, \\ m_c &\sim 1.15 - 1.35 \text{ GeV}, & m_b &\sim 4.0 - 4.4 \text{ GeV}, & m_t &= 174.3 \pm 5.1 \text{ GeV}, \end{aligned} \quad (1)$$

and

$$V_{CKM} = \begin{bmatrix} 0.9742 - 0.9757 & 0.219 - 0.226 & 0.002 - 0.005 \\ 0.219 - 0.225 & 0.9734 - 0.9749 & 0.037 - 0.043 \\ 0.004 - 0.014 & 0.035 - 0.043 & 0.9990 - 0.9993 \end{bmatrix}, \quad (2)$$

where the magnitude range of each matrix element is denoted.

With the one Higgs doublet of the standard model, this pattern (or any other) is certainly allowed, but then Yukawa couplings spanning 5 decades of magnitude are needed. On the other hand, if two Higgs doublets exist with  $v_1 = 174 \text{ GeV}$ , but  $v_2/v_1 \sim 10^{-3} \sim 10^2 \text{ MeV}$ , then Yukawa couplings spanning only 2 decades of magnitude are sufficient. In other words,  $m_{c,b,t}$  are proportional to  $v_1$ , but  $m_{u,d,s}$  are proportional to  $v_2$ . Of course, the hierarchical structure of the 2 vacuum expectation values (VEVs) is yet to be explained. As shown below, this may be attributed to the soft breaking of an assumed U(1) symmetry and is easily implemented if  $\Phi_2$  has  $m^2 > 0$  while  $\Phi_1$  has  $m^2 < 0$  as in the standard model.

The puzzle of quark masses and the charged-current mixing matrix, usually denoted by  $V_{ij}$ , with  $i = u, c, t$  and  $j = d, s, b$ , has received a great deal of continuing attention. One approach is to restrict the number of independent parameters necessary for a general description of all masses and mixing angles, so that a relationship among them may be derived, such as [2]

$$V_{us} = \sqrt{\frac{m_d}{m_s}}. \quad (3)$$

This is usually postulated without recourse to a well-defined symmetry of the Lagrangian nor

the extra particle content required to sustain it [3]. Another shortcoming of this approach is that the mass hierarchy of Eq. (1) remains largely unexplained.

The present approach is different. It looks for a way to understand why  $m_{u,d,s} \ll v = 174$  GeV, i.e. the scale of electroweak symmetry breaking, as well as the pattern of Eq. (2). However, no precise prediction such as Eq. (3) will be made. This approach was used in a radiative scheme some years ago [4], but the model itself is rather complicated. In contrast, the model to be described below is much simpler, requiring only one extra Higgs doublet together with a softly broken global U(1) symmetry.

The U(1) assignments of the 3 generations of quarks and the 2 Higgs doublets are given as follows.

$$(u, d)_L \sim 1, \quad u_R \sim 2, \quad d_R \sim 0; \quad (4)$$

$$(c, s)_L \sim 1, \quad c_R \sim 1, \quad s_R \sim 0; \quad (5)$$

$$(t, b)_L \sim 0, \quad t_R \sim 0, \quad b_R \sim 0; \quad (6)$$

$$(\phi_1^+, \phi_1^0) \sim 0, \quad (\phi_2^+, \phi_2^0) \sim 1. \quad (7)$$

As a result, the *up* quark mass matrix linking  $\overline{(u, c, t)}_L$  to  $(u, c, t)_R$  is given by

$$\mathcal{M}_u = \begin{bmatrix} f_u v_2 & 0 & 0 \\ f_{cu} v_2 & f_c v_1 & 0 \\ 0 & f_{tc} v_2 & f_t v_1 \end{bmatrix}, \quad (8)$$

where  $v_i = \langle \phi_i^0 \rangle$ , and the freedom to rotate among  $(u, d)_L$  and  $(c, s)_L$  has been used to set the  $\bar{u}_L c_R$  element to zero; whereas the *down* quark mass matrix linking  $\overline{(d, s, b)}_L$  to  $(d, s, b)_R$  is given by

$$\mathcal{M}_d = \begin{bmatrix} f_d v_2 & f_{ds} v_2 & f_{db} v_2 \\ 0 & f_s v_2 & f_{sb} v_2 \\ 0 & 0 & f_b v_1 \end{bmatrix}, \quad (9)$$

where the freedom to rotate among the  $(d, s, b)_R$  states has been used to set the 3 lower off-diagonal entries to zero.

Assuming  $v_2 \ll v_1$ , as well as  $f_d \sim f_{ds} \sim f_{db}$  and  $f_s \sim f_{sb}$ , then

$$m_u = f_u v_2, \quad m_d = f_d v_2, \quad m_s = f_s v_2; \quad (10)$$

$$m_c = f_c v_1, \quad m_b = f_b v_1, \quad m_t = f_t v_1. \quad (11)$$

As for  $V_{CKM}$ , the contribution from  $\mathcal{M}_u$  is negligible because they are of order  $(m_u/m_c^2)f_{cu}v_2$  and  $(m_c/m_t^2)f_{tc}v_2$ . Hence

$$V_{cb} \simeq \frac{f_{sb}v_2}{f_b v_1} \simeq \frac{f_{sb}}{f_s} \frac{m_s}{m_b} \simeq \frac{f_{sb}}{f_s} (0.017 - 0.043), \quad (12)$$

$$V_{ub} \simeq \frac{f_{db}v_2}{f_b v_1} \simeq \frac{f_{db}}{f_d} \frac{m_d}{m_b} \simeq \frac{f_{db}}{f_d} (0.001 - 0.002), \quad (13)$$

$$V_{us} \simeq \frac{f_{ds}v_2}{f_s v_2} \simeq \frac{f_{ds}}{f_d} \frac{m_d}{m_s} \simeq \frac{f_{ds}}{f_d} (0.02 - 0.12). \quad (14)$$

Comparing the above with Eq. (2), it is also clear that the Yukawa coupling ratios  $f_{ds}/f_d$ ,  $f_{db}/f_d$ , and  $f_{sb}/f_s$  may all be of order unity. Thus the correct pattern of quark masses and mixing angles is obtained. Obviously, the charged-lepton masses may be treated in the same way, i.e.

$$m_e = f_e v_2, \quad m_\mu = f_\mu v_2, \quad m_\tau = f_\tau v_1. \quad (15)$$

What remains to be shown is how  $v_2 \ll v_1$  can arise naturally.

The most general scalar potential of the 2 assumed scalar doublets is given by

$$\begin{aligned} V = & m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + [\mu_{12}^2 \Phi_1^\dagger \Phi_2 + h.c.], \end{aligned} \quad (16)$$

where the  $\mu_{12}^2$  term breaks the U(1) symmetry softly. The equations of constraint for the VEVs are then

$$v_1 [m_1^2 + \lambda_1 v_1^2 + (\lambda_3 + \lambda_4) v_2^2] + \mu_{12}^2 v_2 = 0 \quad (17)$$

$$v_2 [m_2^2 + \lambda_2 v_2^2 + (\lambda_3 + \lambda_4) v_1^2] + \mu_{12}^2 v_1 = 0. \quad (18)$$

Let  $m_1^2 < 0$ ,  $m_2^2 > 0$ , and  $|\mu_{12}^2| \ll m_2^2$ , then

$$v_1^2 \simeq -\frac{m_1^2}{\lambda_1}, \quad (19)$$

$$v_2 \simeq \frac{-\mu_{12}^2 v_1}{m_2^2 + (\lambda_3 + \lambda_4) v_1^2}. \quad (20)$$

Since the  $\mu_{12}^2$  term breaks the U(1) symmetry, it is natural [5] for it to be small compared to  $m_2^2$ . Thus

$$v_2 \ll v_1 \quad (21)$$

is obtained.

The physical scalar sector of this model consists of a standard-model-like neutral Higgs boson  $H$  (which is mostly  $Re\phi_1^0$ ) and a heavy doublet of mass  $m_2$  approximately. The dominant decays of  $H$  are the same as in the standard model, i.e. into  $\bar{t}t$ ,  $ZZ$ ,  $W^+W^-$ ,  $\bar{b}b$ ,  $\bar{c}c$ , and  $\tau^+\tau^-$ . However, its decays into other final states are modified because they depend on the mixing of  $\Phi_2$  with  $\Phi_1$ . In practice, it will be very difficult to tell the difference because the latter decay modes are very much suppressed.

From Eqs. (8) and (9), it is clear that there are flavor-changing neutral-current (FCNC) interactions in this model, but they are suitably suppressed, as explained below. The matrices  $\mathcal{M}_u$  of Eq. (8) and  $\mathcal{M}_d$  of Eq. (9) are diagonalized according to

$$V_u^\dagger \mathcal{M}_u U_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad (22)$$

$$V_d^\dagger \mathcal{M}_d U_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}, \quad (23)$$

where

$$V_{CKM} = V_u^\dagger V_d, \quad (24)$$

but since  $V_u = 1$  to a very good approximation,  $V_{CKM} \simeq V_d$ , and the  $(d, s, b)_L$  states have to be rotated by  $V_d$  to become mass eigenstates. For example,  $b_L$  in Eq. (9) becomes

$V_{ub}^*d_L + V_{cb}^*s_L + V_{tb}^*b_L$  in the mass-eigenstate basis. Similarly, the  $(d, s, b)_R$  states are rotated by  $U_d$ , i.e.

$$U_d \simeq \begin{bmatrix} V_{ud} & (m_d/m_s)V_{us} & (m_d/m_b)V_{ub} \\ (m_d/m_s)V_{cd} & V_{cs} & (m_s/m_b)V_{cb} \\ (m_d/m_b)V_{td} & (m_s/m_b)V_{ts} & V_{tb} \end{bmatrix}. \quad (25)$$

Thus  $b_R$  becomes  $(m_d/m_b)V_{ub}^*d_R + (m_s/m_b)V_{cb}^*s_R + V_{tb}^*b_R$  in the mass-eigenstate basis.

In the *up* quark sector, the roles of  $V$  and  $U$  are reversed, i.e.

$$U_u \simeq \begin{bmatrix} 1 & f_{cu}v_2/m_c & 0 \\ -f_{cu}v_2/m_c & 1 & f_{tc}v_2/m_t \\ 0 & -f_{tc}v_2/m_t & 1 \end{bmatrix}, \quad (26)$$

and

$$V_u \simeq \begin{bmatrix} 1 & f_{cu}v_2m_u/m_c^2 & 0 \\ -f_{cu}v_2m_u/m_c^2 & 1 & f_{tc}v_2m_c/m_t^2 \\ 0 & -f_{tc}v_2m_c/m_t^2 & 1 \end{bmatrix}, \quad (27)$$

which is the identity matrix to a very good approximation, as mentioned earlier. Thus  $c_L$  becomes  $-(m_u/m_c^2)f_{cu}v_2u_L + c_L + (m_c/m_t^2)f_{tc}v_2t_L$  and  $c_R$  becomes  $-f_{cu}(v_2/m_c)u_R + c_R + f_{tc}(v_2/m_t)t_R$  in the mass-eigenstate basis.

Consider now the phenomenology of the *down* quark sector. Since  $\bar{b}_L b_R$  is the only term which couples to  $\Phi_1$ , if it is replaced by  $\Phi_2$ , there would be no FCNC interactions at all in this sector. Hence all FCNC effects are contained in the term  $f_b \bar{b}_L b_R [\phi_1^0 - (v_1/v_2)\phi_2^0] + h.c.$ , i.e.

$$\begin{aligned} & f_b [V_{ub}V_{tb}^* \bar{d}_L b_R + V_{cb}V_{tb}^* \bar{s}_L b_R + V_{ub}V_{cb}^* (m_s/m_b) \bar{d}_L s_R + V_{tb}V_{cb}^* (m_s/m_b) \bar{b}_L s_R \\ & + V_{cb}V_{ub}^* (m_d/m_b) \bar{s}_L d_R + V_{tb}V_{ub}^* (m_d/m_b) \bar{b}_L d_R] [\phi_1^0 - (v_1/v_2)\phi_2^0] + h.c. \end{aligned} \quad (28)$$

in the mass-eigenstate basis. The most severe constraint on  $m_2$  comes from the  $b \rightarrow s\mu^+\mu^-$  rate through  $\phi_2^0$  exchange, i.e.

$$\frac{\Gamma(b \rightarrow s\mu^+\mu^-)}{\Gamma(b \rightarrow c\bar{\nu}_\mu\mu^-)} \simeq \frac{f_b^2 f_\mu^2 v_1^2}{32G_F^2 m_2^4 v_2^2} < \frac{5.2 \times 10^{-6}}{0.102} = 5.1 \times 10^{-5}, \quad (29)$$

where the experimental  $B^+ \rightarrow K^+ \mu^+ \mu^-$  upper bound has been used for  $b \rightarrow s \mu^+ \mu^-$ , which is of course an overestimate. In other words, the numerical bound of Eq. (29) may not be as small in reality. Using  $f_b = m_b/v_1 = 4.2/174 = 0.024$  and  $f_\mu = m_\mu/v_2$ , Eq. (29) implies

$$m_2 v_2 > 968 \text{ GeV}^2. \quad (30)$$

Thus  $v_2 = 200 \text{ MeV}$  requires  $m_2 > 4.84 \text{ TeV}$ .

The  $K_L - K_S$  mass difference  $\Delta m_K$  gets its main contribution from  $(\bar{d}_L s_R)(\bar{d}_R s_L)$  in this model through  $\phi_2^0$  exchange. Thus

$$\frac{\Delta m_K}{m_K} \simeq \frac{B_K f_K^2 v_1^2}{3m_2^2 v_2^2} f_b^2 |V_{ub} V_{cb}|^2 \frac{m_s m_d}{m_b^2}. \quad (31)$$

Using  $f_K = 114 \text{ MeV}$ ,  $B_K = 0.4$ ,  $|V_{ub}| = 0.0035$ ,  $|V_{cb}| = 0.040$ ,  $m_s = 125 \text{ MeV}$ ,  $m_d = 7 \text{ MeV}$ , and Eq. (30), this contribution is then less than  $3.1 \times 10^{-20}$ , which is certainly negligible compared against the experimental value of  $7.0 \times 10^{-15}$ .

Similarly, the  $\Delta m_{B^0}$  and  $\Delta m_{B_s^0}$  contributions are

$$\frac{\Delta m_{B^0}}{m_{B^0}} \simeq \frac{B_B f_B^2 v_1^2}{3m_2^2 v_2^2} f_b^2 |V_{ub} V_{tb}|^2 \frac{m_d}{m_b}, \quad (32)$$

and

$$\frac{\Delta m_{B_s^0}}{m_{B_s^0}} \simeq \frac{B_B f_B^2 v_1^2}{3m_2^2 v_2^2} f_b^2 |V_{cb} V_{tb}|^2 \frac{m_s}{m_b}. \quad (33)$$

Using  $f_B = 170 \text{ MeV}$ ,  $B_B = 1.0$ ,  $|V_{tb}| = 1$ , and the other parameter values as before, these contributions are respectively less than  $3.7 \times 10^{-15}$  and  $8.5 \times 10^{-12}$ , to be compared against the experimental value of  $5.9 \times 10^{-14}$  for the former and the experimental lower bound of  $1.3 \times 10^{-12}$  for the latter.

In the case of  $D^0 - \bar{D}^0$  mixing, the main contribution comes from  $(\bar{c}_L u_R)(\bar{c}_R u_L)$ , i.e.

$$\frac{\Delta m_{D^0}}{m_{D^0}} \simeq \frac{B_D f_D^2 v_1^2}{3m_2^2} f_c^2 f_{cu}^2 \frac{m_u}{m_c^3}. \quad (34)$$

Using  $f_D = 150$  MeV,  $B_D = 0.8$ ,  $m_c = 1.25$  GeV,  $f_c = f_{cu} = m_c/v_1 = 0.0072$ , and  $m_u = 4$  MeV, this contribution is then  $1.0 \times 10^{-15}$  (1 TeV/ $m_2$ )<sup>2</sup>, well below the experimental upper bound of  $2.5 \times 10^{-14}$ .

Other FCNC processes are also suppressed. For example,

$$\Gamma(K_L \rightarrow \mu^+ \mu^-) \simeq \frac{f_K^2 m_K^3}{64\pi} \frac{f_b^2 m_\mu^2 v_1^2}{m_2^4 v_2^4} |V_{ub} V_{cb}|^2 \frac{m_s^2}{m_b^2}. \quad (35)$$

Using the previously chosen values for all the parameters, this contribution is less than  $3.1 \times 10^{-29}$ , well below the experimental value of  $9.2 \times 10^{-26}$  GeV. As for  $K_L \rightarrow e^+ e^-$ , it is further suppressed by  $m_e^2/m_\mu^2$ , resulting in a contribution of less than  $7.2 \times 10^{-34}$ , which is even more negligible compared to the experimental value of  $1.1 \times 10^{-28}$  GeV. Finally, the  $b \rightarrow s\gamma$  rate receives a contribution proportional to  $|f_b V_{cb}|^2/m_2^2$ , which would be competitive with the standard model if  $f_b$  were of order unity and  $m_2$  of order  $M_W$ , but since  $f_b = 0.024$  and  $m_2 \gg M_W$  in this model, it is again negligible.

There is also a contribution from  $\Phi_2$  to the muon anomalous magnetic moment [6]. It is easily calculated to be

$$\Delta a_\mu = \frac{f_\mu^2}{16\pi^2} \frac{m_\mu^2}{m_2^2} \left( \ln \frac{m_2^2}{m_\mu^2} - 1 \right), \quad (36)$$

which is of the order  $10^{-11}$  or less, and thus negligible. However, the present model can be extended to allow for neutrino masses using a leptonic Higgs doublet [7], then the possible observed discrepancy in  $\Delta a_\mu$  may be explained [8], but a nearly degenerate neutrino mass matrix is required. The extra contributions from  $\Phi_2$  to the oblique parameters  $S, T, U$  in precision electroweak measurements are all suppressed by  $\lambda_4 v_1^2/m_2^2$  and do not upset the excellent fit of the standard model.

In summary, a new realization of the generation structure of quarks and leptons has been presented in this paper, as given by Eqs. (4) to (7). The one extra scalar doublet is heavy with  $m^2 > 0$ . Typical values are  $m_2 \sim$  few TeV with  $v_2 \sim$  fraction of a GeV, whereas  $\Phi_1$



has  $m^2 < 0$ , resulting in  $v_1 = 174$  GeV and  $m_H = 115$  GeV or greater. This is accomplished by a softly broken U(1) symmetry with  $|\mu_{12}|^2/m_2^2 \sim 10^{-3}$ . The pattern of the observed quark masses (with  $m_{u,d,s}$  from  $v_2$  and  $m_{c,b,t}$  from  $v_1$ ) and the corresponding charged-current mixing matrix ( $V_{CKM}$ ) is realized without severely hierarchical Yukawa couplings. All FCNC effects of this model are suitably suppressed if  $m_2 v_2 > 968$  GeV<sup>2</sup> and do not change the good agreement of the standard model with present data. The standard-model-like neutral Higgs boson of this model has dominant decays identical to those of the standard model. To distinguish the two models, the discovery of  $b \rightarrow s\mu^+\mu^-$  would help, but finding the extra Higgs doublet  $\Phi_2$  would be more decisive. If  $m_2$  is nearer 1 TeV than 5 TeV, then future high-energy accelerators such as the Large Hadron Collider (LHC) will have a reasonable chance of doing it.

This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.

## References

- [1] Particle Data Group: D. E. Groom *et al.*, Eur. Phys. J. **C15**, 1 (2000).
- [2] S. Weinberg, Ann. N. Y. Acad. Sci. **38**, 185 (1977).
- [3] There are some exceptions. See for example E. Ma, Phys. Rev. **D43**, R2761 (1991); L. Lavoura, Phys. Rev. **D61**, 077303 (2000).
- [4] E. Ma, Phys. Rev. Lett. **64**, 2866 (1990).
- [5] G. 't Hooft, in *Recent Developments in Gauge Theories: Proceedings of the NATO Advanced Study Institute (Cargese, 1979)*, eds. G. 't Hooft *et al.*, (Plenum, New York, 1980).
- [6] H. N. Brown *et al.*, Phys. Rev. Lett. **86**, 2227 (2001).
- [7] E. Ma, Phys. Rev. Lett. **86**, 2502 (2001).
- [8] E. Ma and M. Raidal, hep-ph/0102255.